 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - MATHEMATICS

THIRD SEMESTER – NOVEMBER 2012

# MT 3811 - COMPLEX ANALYSIS

Date : 03/11/2012 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

**Answer all the questions:**

1. a) Prove that if using Leibniz’s rule.

OR

b) State and prove Liouville’s theorem. (5)

c) State and prove first version of Cauchy’s integral formula.

OR

d) State and prove the homotopic version of Cauchy’s theorem (15)

2. a) State and prove Hadamard’s three circles theorem.

OR

b) Define a convex function and prove that a function is convex if and

only if the set is a convex set. (5)

c) State and prove Goursat’s theorem.

OR

d) State and prove Arzela Ascoli theorem. (15)

3. a) Let , for all . Then prove that converges to a complex number different from zero if and only if converges.

OR

b) Show that in the usual notation. (5)

c) (i) If and then prove that .

(ii) Prove that .

(iii) State and prove Gauss’s Formula. (5+5+5)

OR

d) (i) State and prove Bohr-Mollerup theorem.

(ii) Prove that (a) converges to in and (b) if then for all . (8+7)

4. a) State and prove Jensen’s formula.

OR

b) Let be a rectifiable curve and let *K* be a compact set such that . If f is a continuous function on and then prove that there is a rational function having all its poles on and such that for all *z* in *K*.

(5)

c) State and prove Mittag-Leffler’s theorem. (15)

OR

d) State and prove Hadamard’s Factorization theorem.

(15)

5. a) Prove that any two bases of a same module are connected by a unimodular transformation.

OR

b) Show that and it is an odd function. (5)

c) (i) Prove that the zeros and poles of an elliptic function satisfy .

(ii) Prove that . (7+8)

OR

d) (i) Show that

(ii) State and prove the addition theorem for the Weierstrass -function. (7+8)

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